1) Let X_1, X_2, X_3, X_4 and $X_5 \sim$ iid Gamma(2, 5).The mean of Gamma(α, β) is $\frac{\alpha}{\beta}$. Find the mean of the distribution $X_1 + X_2 + X_3 + X_4 + X_5$.

Sum of 2 iid gamma random variables is,

$$\text{Gamma}(\alpha_1+\alpha_2, \beta)$$

where,

$X \sim \text{Gamma}(\alpha_1, \beta)$

$Y \sim \text{Gamma}(\alpha_2, \beta)$

$$\Rightarrow E[X_1 + X_2 + X_3 + X_4 + X_5] = E[\text{Gamma}(2+2+2+2+2, 5)]$$

$$\Rightarrow \frac{10}{5} = \underline{\underline{2}}$$

2) Let X_1, X_2 and $X_3 \sim$ iid Normal(0, 9). Find the mean of $X_1^2 + X_2^2 + X_3^2$.

We know,

Square of iid $\text{Normal}(0, \sigma^2) = \text{Gamma}\left(\frac{1}{2}, \frac{1}{2\sigma^2}\right)$

$$\Rightarrow X_1^2 = \text{Gamma}\left(\frac{1}{2}, \frac{1}{2\times 9}\right)$$

$$E[X_1^2 + X_2^2 + X_3^2] = E\left[\text{Gamma}\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}, \frac{1}{18}\right)\right] \qquad [X_1^2 = X_2^2 = X_3^2]$$

$$\Rightarrow \frac{3/2}{1/18} = \underline{\underline{27}}$$

3) Let X_1, X_2, X_3 and $X_4 \sim$ iid Normal(0, 4). Find the variance of $X_1^2 + X_2^2 + X_3^2 + X_4^2$.

We know,

$$X_1^2 = \text{Gamma}\left(\frac{1}{2}, \frac{1}{2\sigma^2}\right)$$

$$\Rightarrow \text{Gamma}\left(\frac{1}{2}, \frac{1}{2\times 4}\right) \qquad [\sigma^2 = 4]$$

$$X_1^2 + X_2^2 + X_3^2 + X_4^2 = \text{Gamma}\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}, \frac{1}{8}\right)$$

$$\Rightarrow \text{Gamma}\left(2, \frac{1}{8}\right)$$

$$\Rightarrow \text{Var}(\text{Gamma}(2, 1/8)) = \frac{2}{(1/8)^2} = \underline{\underline{128}}$$