$F_Z(0.2617) = 0.603, F_Z(1.6) = 0.9452, F_Z(1.5) = 0.933$

1) The average life of a coffee machine is 5 years, with a standard deviation of 1 year. Assuming that the lives of these machines follow approximately a normal distribution, find the probability that the mean life of a random sample of 16 such machines falls between 4.6 and 5.4 years. Enter your answer correct to two decimals.

Given,

$\mu = 5$ | $n = 16$

$\sigma = 1$

Let mean of 16 coffee machines be $\overline{X}$

$\Rightarrow \overline{X} = \dfrac{x_1 + x_2 + x_3 - - - - + x_{16}}{16}$

Also $Y = X_1 + X_2 + X_3 - - - - + X_{16}$

$\Rightarrow \overline{X} = \dfrac{Y}{16}$

Find : $P(4.6 < \overline{X} < 5.4)$

$\Rightarrow P\left(4.6 < \dfrac{Y}{16} < 5.4\right)$

$\Rightarrow P\left(73.6 < Y < 86.4\right)$

By Formula,

$Z = \dfrac{Y - n\mu}{\sqrt{n}\,\sigma}$

$\Rightarrow Z = \dfrac{Y - (16)(5)}{\sqrt{16} \times 1}$

$\Rightarrow Z = \dfrac{Y - 80}{4}$

$\Rightarrow$ To get $P(Z)$ within a range

$\Rightarrow P\left(73.6 - 80 < Y - 80 < 86.4 - 80\right)$

$\Rightarrow P\left(-6.4 < Y - 80 < 6.4\right)$

$\Rightarrow P\left(\dfrac{-6.4}{4} < \dfrac{Y-80}{4} < 6.4\right)$

$\Rightarrow P\left(-1.6 < \underbrace{\dfrac{Y-80}{4}}_{Z} < 1.6\right)$

We know,

$F_Z(a) + F_Z(-a) = 1$

$\Rightarrow P(-1.6 < Z < 1.6)$

$\Rightarrow F_Z(1.6) - F_Z(-1.6)$

$\Rightarrow F_Z(1.6) - (1 - F_Z(1.6))$

$\Rightarrow 2F_Z(1.6) - 1$

$\Rightarrow 2(0.9452) - 1$

$\Rightarrow \underline{\underline{0.89}}$

2) The random variable X representing the number of cherries in a cherry puff has the following probability distribution:

X	4	6	8	10
$P(X=x)$	0.3	0.1	0.4	0.2

Table 7.6.1: PMF of X

Find the probability that the average number of cherries in 64 cherry puffs will be less than 7. Enter your answer correct to two decimals.

Given,

$n = 64$

$E[X] = 4\times0.3 + 6\times0.1 + 8\times0.4 + 10\times0.2$

$\Rightarrow 1.2 + 0.6 + 3.2 + 2$

$\Rightarrow 7$

$[\therefore \mu = 7]$

$\Rightarrow Var(X) = E[X^2] - E[X]^2$

$\Rightarrow E[X^2] = 16\times0.3 + 36\times0.1 + 64\times0.4 + 100\times0.2$

$= 54$

$\Rightarrow Var(X) = 54 - (7)^2 = 5$

$\left[\therefore \sigma^2 = 5 \Rightarrow \sigma = \sqrt{5}\right]$

Now, $P(\bar{X} < 7)$

$\Rightarrow P(Y < 7\times64)$

$\Rightarrow P(Y < 448)$

$\Rightarrow P\left(\frac{Y - n\mu}{\sqrt{n}\sigma} < \frac{448 - 448}{\sqrt{n}\sigma}\right)$

$\Rightarrow P(Z < 0)$

$\Rightarrow F_Z(0) = \frac{1}{2} = \underline{\underline{0.5}}$

3) An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed, with mean equal to 600 hours and a standard deviation of 50 hours. Find the probability that a random sample of 25 bulbs will have an average life of less than 615 hours. Enter your answer correct to two decimals.

Given,

$X \sim Normal(\underbrace{600}_{\mu}, \underbrace{(50)^2}_{\sigma^2})$

$n = 25$

X = Length of life

$P(\bar{X} < 615)$

$\Rightarrow P(Y < 615\times25)$

$\Rightarrow P\left(\frac{Y - n\mu}{\sqrt{n}\sigma} < \frac{615\times25 - 25\times600}{5\times50}\right)$

$\Rightarrow P\left(Z < \frac{\cancel{25}^{5}(15)}{\cancel{5}\times\cancel{50}_{10}}\right)$

$\Rightarrow P(Z < 1.5) = F_Z(1.5) = \underline{\underline{0.933}}$

4) The number of accidents in a certain city is modeled by a Poisson random variable with an average rate of 9 accidents per day. Suppose that the number of accidents on different days are independent. Use the central limit theorem to find the probability that there will be more than 3300 accidents in a certain year. Assume that there are 365 days in a year. Enter your answer correct to two decimals.

X = No. of accidents in a day

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

We know,

$E[X] = \lambda$, $Var(X) = \lambda$

Here, $\lambda = 9$

$\therefore \mu = \sigma^2 = 9$

$P(X > 3300)$

$$\Rightarrow P\left(\frac{X - n\mu}{\sqrt{n}\,\sigma} > \frac{3300 - n\mu}{\sqrt{n}\,\sigma}\right)$$

$$\Rightarrow P\left(Z > \frac{3300 - 365\times 9}{\sqrt{365}\,(3)}\right) \qquad [n = 365]$$

$\Rightarrow 1 - F_Z(5/\sqrt{365})$

$\Rightarrow 1 - F_Z(0.26107)$

$\Rightarrow 1 - 0.603$

$\Rightarrow \underline{\underline{0.397}}$