1) Let $(X, Y) \sim \text{Uniform}(D)$, where $D := [0,2] \times [2,4]$. Choose the correct option(s) from the following:

$X \sim \text{Uniform}[0,2]$.

$Y \sim \text{Uniform}[2,4]$

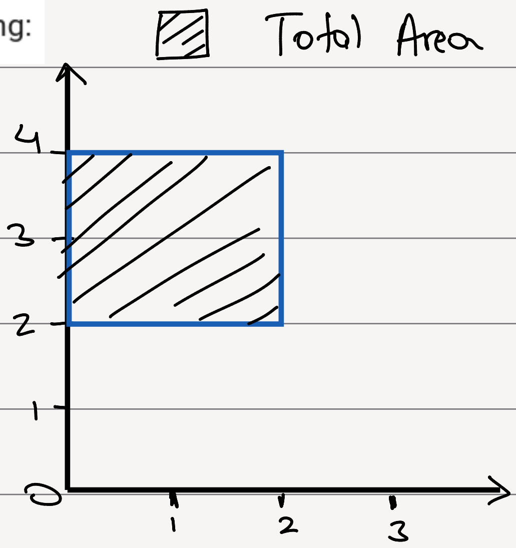
$f_X(x) = 2, 0 < x < 2$

$f_X(x) = \frac{1}{2}, 0 < x < 2$.

$$D := \underbrace{[0,2]}_{x} \times \underbrace{[2,4]}_{y}$$

$$\Rightarrow X \sim \text{Uniform}[0,2]$$

$$Y \sim \text{Uniform}[2,4]$$



$$\text{Total Area} = l \times b$$

$$\Rightarrow (2-0) \times (4-2)$$

$$\Rightarrow 4$$

$$\Rightarrow \text{Density} = \frac{1}{|\text{Area}|} = \frac{1}{4}$$

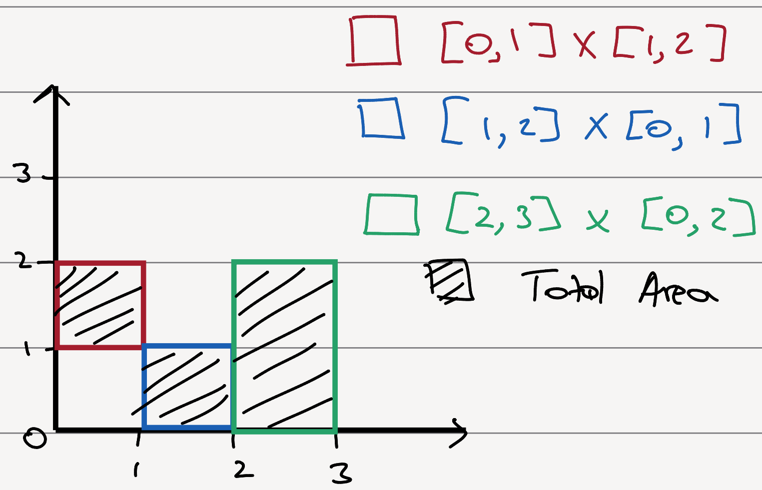
$$f_X(x) = \int_2^4 \frac{1}{4} \cdot dy = \frac{1}{4}[y]_2^4 = \frac{1}{4}[4-2] = \frac{1}{2}$$

2) Let $(X, Y) \sim \text{Uniform}(D)$, where $D := [0,1] \times [1,2] \cup [1,2] \times [0,1] \cup [2,3] \times [0,2]$. Choose the correct option(s) from the following:

$$\text{Total Area} = (1-0) \times (2-1) + (2-1) \times (1-0) + (3-2) \times (2-0)$$

$$\Rightarrow 1 + 1 + 2$$

$$\Rightarrow 4$$



$$\Rightarrow \text{Density} = \frac{1}{|\text{Area}|} = \frac{1}{4}$$

Let X and Y be continuous random variables with joint density function

$$f_{XY}(x,y) = \begin{cases} \frac{1}{4} & \text{for } 0 < x < \frac{y}{2}, 0 < y < 4 \\ 0 & \text{otherwise} \end{cases}$$

3) Find the marginal density of Y.

$$f_Y(y) = \int_0^{y/2} \frac{1}{4} \cdot dx$$

$$\Rightarrow \frac{1}{4}[x]_0^{y/2} = \frac{y}{8}$$

$$\therefore f_Y(y) = \begin{cases} y/8 & 0 < y < 4 \\ 0 & \text{otherwise} \end{cases}$$

4) Find the marginal density of X.

We know,

$0 < x < y/2$

$\Rightarrow 0 < 2x < y$ also $0 < y < 4$

$\Rightarrow$ Y lies between $2x$ and 4

$\Rightarrow f_X(x) = \int_{2x}^{4} 1/4 \cdot dy$

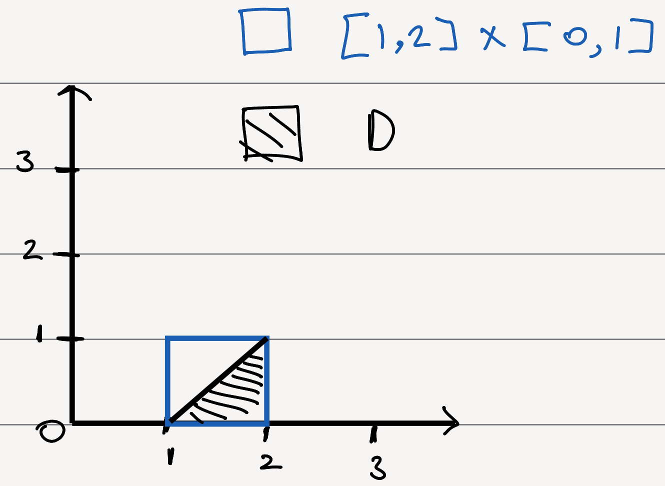
$\Rightarrow \frac{1}{4}[y]_{2x}^{4} = \frac{1}{4}[4-2x] = 1 - \frac{x}{2}$

$$\therefore f_X(x) = \begin{cases} 1 - \frac{x}{2} & 0 < x < 4/2 \\ 0 & \text{otherwise} \end{cases}$$

5) Let $(X, Y) \sim \text{Uniform}(D)$, where $D := \{(x, y) : x - y \geq 1, 1 < x < 2, y > 0\}$. Choose the correct option(s) from the following:

$x - y \geq 1 \quad , \quad 1 < x < 2 \quad , \quad y > 0$

As x lies between 1 and 2, maximum value of y can be 1, else the condition $x - y \geq 1$ wont satisfy.



$\Rightarrow$ Conditions of joint PDF are

$x - y = 1 \quad , \quad 1 < x < 2 \quad , \quad 0 < y < 1$

Area of shaded region $= \frac{1}{2} \times 1 \times 1$

$\Rightarrow 1/2$

Density $= \frac{1}{|Area|} = \frac{1}{1/2} = 2$

The maximum value x can take is 2.

The minimum value x can take is 1, but $x - y = 1$ should be satisfied. $x = y + 1$ is the lowest value x can take when $y = 0$.

$$\Rightarrow f_Y(y) = \int_{x=1+y}^{x=2} 2 \cdot dx$$

$$\Rightarrow 2\,[x]_{1+y}^{2} = 2\,[2-(1-y)] = \underline{\underline{2\,[1+y]}}$$

The minimum value y can take is 0.
The maximum value y can take is 1, but $x-y=1$ should be satisfied. $y = x-1$ is the highest value y can take when $x=2$.

$$\Rightarrow f_X(x) = \int_{y=0}^{y=x-1} 2 \cdot dy$$

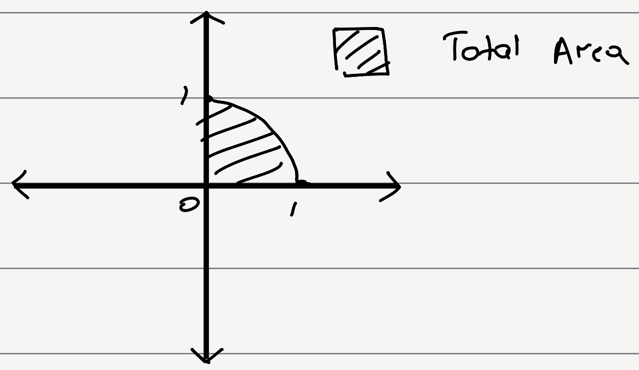
$$\Rightarrow 2\,[y]_0^{x-1} = \underline{\underline{2\,[x-1]}}$$

6) Let $(X,Y) \sim \text{Uniform}(D)$, where $D := \{x^2+y^2 \le 1, x>0, y>0\}$ Choose the correct option(s) from the following:

Total Area $= (\pi r^2)/4$

$\Rightarrow (\pi (1)^2)/4$

$\Rightarrow \frac{\pi}{4}$



Density $= \frac{1}{|\text{Area}|} = \frac{4}{\pi}$

$$f_X(x) = \int_{y=0}^{y=\sqrt{1-x^2}} \frac{4}{\pi} \cdot dy$$

$$\Rightarrow \frac{4}{\pi}\,[y]_0^{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{4}{\pi}\,[\sqrt{1-x^2}]$$

$$f_Y(y) = \int_{x=0}^{x=\sqrt{1-y^2}} \frac{4}{\pi} \cdot dx$$

$$\Rightarrow \frac{4}{\pi}\,[x]_0^{\sqrt{1-y^2}}$$

$$\Rightarrow \frac{4}{\pi}\,[\sqrt{1-y^2}]$$