1) Let $X \sim$ Uniform$[2, 6]$. Find the value of $P(1 < X < 5)$. (Write your answer correct to two decimal places.)

$$f_X(x) = \begin{cases} \frac{1}{6-2} & 2 < x < 6 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow \int_2^5 \frac{1}{4}$$

$$\Rightarrow \frac{1}{4}\left[x\right]_2^5$$

$$\Rightarrow \frac{3}{4} = \underline{\underline{0.75}}$$

2) Let $X \sim$ Uniform$[a, b]$. If $P(X \leq a + 2) = 0.2$, what will be the value of $P(X \geq b-2)$?

$$f_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

Given,

$$P(X \leqslant a+2) = 0.2$$

$$\Rightarrow \int_a^{a+2} f_X(x) = 0.2$$

$$\Rightarrow \frac{1}{b-a}\left[x\right]_a^{a+2} = 0.2$$

$$\Rightarrow \frac{2}{b-a} = 0.2$$

$$\Rightarrow b-a = 10 \quad \text{——} \quad (1)$$

$$\Rightarrow b = 10 + a$$

Subtracting 2 from both sides

$$\Rightarrow \underline{b-2 = 8+a} \quad \text{——} \quad (2)$$

$$P(X \geqslant b-2) = 1 - P(X \leqslant b-2)$$

$$P(X \leqslant b-2) = \int_a^{b-2} f_X(x)$$

$$\Rightarrow \int_a^{8+a} \frac{1}{b-a} \quad [\text{From } (2)]$$

$$\Rightarrow \int_a^{8+a} \frac{1}{10} \quad [\text{From } (1)]$$

$\Rightarrow \quad 0.1 \, [x]_a^{8+a}$

$\Rightarrow \quad 0.8$

$\therefore \; P(X \geq b-2) = 1 - P(X \leq b-2)$

$\Rightarrow 1 - 0.8$

$\Rightarrow \underline{\underline{0.2}}$

3) The amount of milk produced every day by a dairy is uniformly distributed between 100 litres and 120 litres. What is the probability that the amount of milk produced is more than 115 litres given that on that day, more than 110 litres of milk was produced?

Given,

$X \sim \text{Uniform}[100, 120]$

$$P(X > 115 \mid X > 110) = \frac{P(X > 115 \text{ and } X > 110)}{P(X > 110)}$$

$$\Rightarrow \frac{P(X > 115)}{P(X > 110)}$$

$$\Rightarrow \frac{1 - P(X \leq 115)}{1 - P(X \leq 110)} \quad \text{--- } (1)$$

$$f_X(x) = \begin{cases} \dfrac{1}{120-100} , & 100 < x < 120 \\ 0 , & \text{otherwise} \end{cases}$$

$$\Rightarrow P(X \leq 115) = \int_{100}^{115} \frac{1}{20}$$

$$\Rightarrow \frac{15}{20}$$

$$P(X \leq 110) = \int_{100}^{110} \frac{1}{20}$$

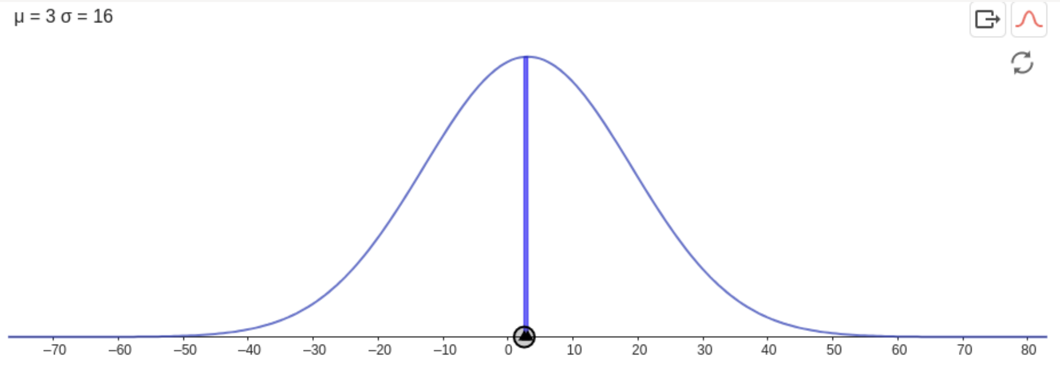
$$\Rightarrow \frac{10}{20}$$

From (1),

$$\Rightarrow \frac{1 - 15/20}{1 - 10/20}$$

$$\Rightarrow \frac{\frac{5}{20}}{\frac{10}{20}} = \underline{\underline{0.5}}$$

4) Let $X \sim$ Normal$(3, 16)$. Then, around which value X is most likely to occur?



This is a normal distribution graph, $\mu = 3$ and $\sigma^2 = 16$

A normal curve takes the mean value with the highest probability.

$\Rightarrow$ Most likely value to occur $= \underline{\underline{3}}$

5) Let $X \sim$ Exp(5). Find the value of $P(X \leq 10)$.

$$F_X(x) = \begin{cases} 0, & x \leq 0 \\ 1 - \exp(-5x), & x > 0 \end{cases}$$ [Formula]

$$\Rightarrow F_X(10) = 1 - e^{-5\times 10}$$

$$\Rightarrow \underline{\underline{1 - e^{-50}}}$$

6) Let $X \sim \text{Exp}(2)$. Find the value of $P(3 \leq X \leq 5)$.

$$F_X(x) = \begin{cases} 0, & x \leq 0 \\ 1 - \exp(-2x), & x > 0 \end{cases}$$

$$\Rightarrow P(3 \leq X \leq 5) = F_X(5) - F_X(3)$$

$$\Rightarrow (1 - e^{-10}) - (1 - e^{-6})$$

$$\Rightarrow \underline{\underline{e^{-6} - e^{-10}}}$$

7) Let $X \sim$ Normal$(50, 25)$. Find $P(X \geq 50)$.

$\mu = 50$, $\sigma^2 = 25$

$\Rightarrow \sigma = 5$

$$P(X \geq 50) = 1 - P(X \leq 50)$$

Solving for $P(X \leq 50)$,

$$\Rightarrow P\left(\frac{X - \mu}{\sigma} \leq 50\right)$$

$\Rightarrow P\left(Z \leq \frac{50-50}{5}\right)$

$\Rightarrow F_Z(0)$

$\Rightarrow 0.5$ [From standard normal distribution table]

$\therefore P(X \geq 50) = 1 - P(X \leq 50) = 1 - F_Z(0) = 1 - 0.5 = \underline{\underline{0.5}}$

8) Let $X \sim$ Normal$(50, 100)$. Find $P(45 < X < 62)$. (Assume that F_Z denote the CDF of standard normal distribution.)

$\mu = 50, \quad \sigma = \sqrt{100}$

$P(45 < x < 62)$

$\Rightarrow P\left(\frac{45-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{62-\mu}{\sigma}\right)$

$\Rightarrow P\left(\frac{45-50}{10} < Z < \frac{62-50}{10}\right)$

$\Rightarrow P(-0.5 < z < 1.2)$

$\Rightarrow \underline{\underline{F_Z(1.2) - F_Z(-0.5)}}$