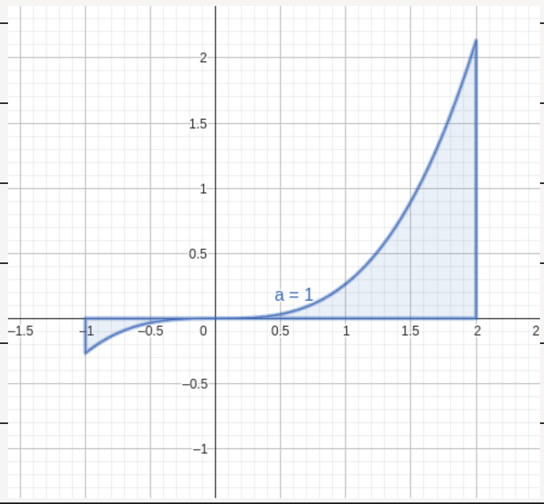
3) Consider a function $f : R \to R$ such that

$$f(x) = \begin{cases} \frac{4}{15}x^3 & -1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$



$\Rightarrow \int_a^b f(x)$

$\Rightarrow \int_{-1}^{2} \frac{4}{15} x^3$

$\Rightarrow \left[\frac{x^4}{4}\right]_{-1}^{2} \times \frac{4}{15}$

$\Rightarrow (2^4 - (-1)^4) \times \frac{1}{15}$

$\Rightarrow 1$

This is not a valid density function because x can take negative values.

$\Rightarrow f_x(x)$ will also be negative in the range of $[-1, 0]$.

4) Consider a function $f : R \to R$ such that

$$f(x) = \begin{cases} cx^3 & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

Which of the following combinations of a,b and c will make f a valid density function?

- ☐ $a = -2, b = 1$ and $c = \frac{4}{15}$
- ✓ $a = 1, b = 2$ and $c = \frac{4}{15}$
- ☐ $a = 1, b = 2$ and $c = \frac{1}{15}$
- ✓ $a = 2, b = 3$ and $c = \frac{4}{65}$

$\int_a^b f(x) = 1$

$\Rightarrow \int_a^b cx^3 = 1$

$\Rightarrow \frac{1}{c} = \left[\frac{x^4}{4}\right]_a^b$

$\Rightarrow \frac{4}{c} = b^4 - a^4$

Check each option which satisfies this condition.

5) The probability density function of a random variable X is given as

$$f_X(x) = \begin{cases} 0.1 & 0 \leq x < 1 \\ cx & 1 \leq x < 2 \\ 0.3 & 2 \leq x < 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of $P(1 < X < 2.5)$.

$$\int f_X(x) = 1$$

$$\Rightarrow \int_0^1 0.1 + \int_1^2 cx + \int_2^3 0.3 = 1$$

$$\Rightarrow 0.1[x]_0^1 + c\left[\frac{x^2}{2}\right]_1^2 + 0.3[x]_2^3 = 1$$

$$\Rightarrow \frac{3c}{2} + 0.4 = 1$$

$$\Rightarrow c = \cancel{0.6}^{0.2} \times \frac{2}{\cancel{3}}$$

$$\Rightarrow c = 0.4$$

$$P(1 < X < 2.5) = \int_1^2 0.4x + \int_2^{2.5} 0.3$$

$$\Rightarrow 0.4\left[\frac{x^2}{2}\right]_1^2 + 0.3[x]_2^{2.5}$$

$$\Rightarrow 0.4 \times \frac{3}{2} + 0.3 \times 0.5$$

$$\Rightarrow 0.6 + 0.15$$

$$\Rightarrow 0.75$$

Consider the following probability density function f_X of a random variable X to answer the questions **(6), (7)** and **(8)**.

$$f_X(x) = \begin{cases} kx & 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

7) Find the value of $P(0.5 \le X \le 0.8)$. (Write your answer correct to two decimal places.)

$$\int_0^1 f_X(x) = 1$$

$$\Rightarrow \int_0^1 kx = 1$$

$$\Rightarrow \left[\frac{x^2}{2}\right]_0^1 k = 1$$

$$\Rightarrow k = 2$$

$$P(0.5 < X < 0.8) = \int_{0.5}^{0.8} 2x$$

$$\Rightarrow 2\left[\frac{x^2}{2}\right]_{0.5}^{0.8}$$

$$\Rightarrow 0.64 - 0.25$$

$$\Rightarrow 0.39$$

8) Find the value of $F(0.5)$ where F is the CDF of X. (Write your answer correct to two decimal places.)

$$F(0.5) = \int_0^{0.5} f_X(x)$$

$$\Rightarrow \int_0^{0.5} 2x$$

$$\Rightarrow 2\left[\frac{x^2}{2}\right]_0^{0.5}$$

$$\Rightarrow \underline{\underline{0.25}}$$