1) Which of the following option(s) is (are) correct about a continuous random variable X?

- [x] $P(X = x) = 0$ for any real number x. (Property)
- [ ] $P(X = x) = 0$ only for some real number x.
- [x] CDF (F) of X is a non decreasing continuous function. [CDF of continuous random variable is always continuous.]
- [ ] If $P(X = x) = 0$, x is not in the range of X.

2) Which of the following functions **cannot** represent the CDF of a continuous random variable? (Note: All the functions are non decreasing functions.)

a) $F(x) = \begin{cases} 0 & x < -2 \\ \frac{1}{3}(0.5x + 1.4) & -2 \le x < 1 \\ 0.7 & 1 \le x < 2 \\ 0.3x + 0.1 & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$ ✓

b) $F(x) = \begin{cases} 0 & x < -2 \\ \frac{1}{3}(0.7x + 1.4) & -2 \le x < 1 \\ 0.7 & 1 \le x < 2 \\ 0.3x + 0.1 & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$

c) $F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4} + \frac{1}{2}(1 - e^{-x}) & 0 \le x < 1 \\ \frac{1}{2} + \frac{1}{2}(1 - e^{-x}) & x \ge 1 \end{cases}$ ✓

d) $F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}(1 - e^{-x}) & x \ge 0 \end{cases}$ ✓

a) Testing continuity at $x=1$,

$\frac{1}{3}(0.5(1) + 1.4) = 0.7$

$\Rightarrow \frac{2.9}{3} \neq 0.7$

$\Rightarrow$ This function is not continuous.

b) Testing continuity at $x=1$,

$\Rightarrow \frac{1}{3}(0.7(1) + 1.4) = 0.7$

$\Rightarrow \frac{2.1}{3} = 0.7$

$\Rightarrow 0.7 = 0.7$

Testing continuity at $x=2$,

$\Rightarrow 0.7 = 0.3(2) + 1$

$\Rightarrow 0.7 = 0.7$

$\Rightarrow$ This function is continuous and a valid CDF.

c) Testing continuity at $x=0$,

$\Rightarrow 0 = \frac{1}{4} + \frac{1}{2}(1 - e^{-0})$

$\Rightarrow 0 \neq 1/4$

$\Rightarrow$ This function is not continuous.

d) Testing continuity at $x=0$,

$\Rightarrow 0 = \frac{1}{2}(1 - e^{-0})$

$\Rightarrow 0 = 0$

This function is continuous but it is not a valid CDF, as it never reaches the probability 1.

Use the following information to answer the questions (3), (4) and (5). The cumulative distribution function of a continuous random variable X is given as:

$$F(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{2}(x+1)^2 & -1 \le x < 0 \\ 1 - \frac{(1-x)^2}{2} & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

3) Find the value of $P(X = 0)$.

$F(X = x) = 0$ for a valid CDF.

$\Rightarrow P(X=0) = 0.$

4) Find the value of $P(-3 \le X \le 0.5)$ (write your answer up to two decimal places.)

$\Rightarrow P(-3 \leqslant X \leqslant 0.5) = F_x(0.5) - F_x(-3)$

$F_x(-3) = 0$ $\quad$ [As $P(X < -1) = 0$]

$F_x(0.5) = 1 - \frac{(1-0.5)^2}{2} = 0.875$

$\Rightarrow P(-3 \leqslant X \leqslant 0.5) = 0.875 - 0 = 0.875$

5) Find the value of $P(X \ge 0)$.

$\Rightarrow P(X \geqslant 0) = 1 - \frac{(1-0)^2}{2} = \frac{1}{2} = 0.5$

Consider the following CDF of a random variable X to answer the questions (6), (7) and (8).

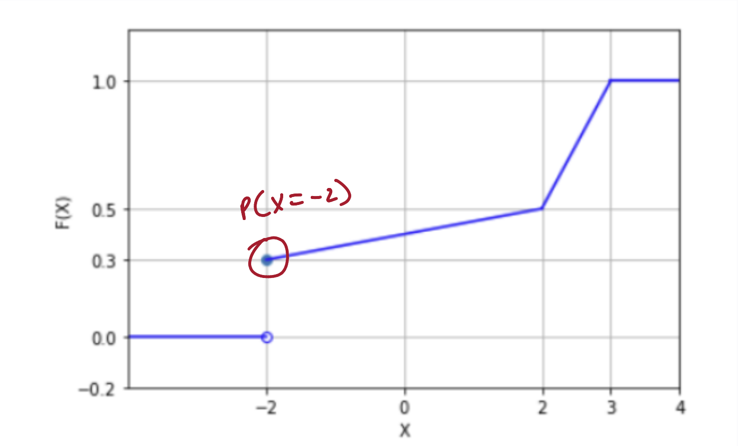

Figure 4.4.1: CDF of X

6) Find the value of $P(X = -2)$.

$P(X = -2) = 0.3$

7) Find the value of $P(0 \le X \le 2.5)$.(write your answer correct to two decimal places.)
(Hint: Try to find the equations of lines.)

$0.3 = m(-2) + c$ $\qquad\qquad$ $0.5 = m(2) + c$

$0.5 = m(2) + c$ $\qquad\qquad$ $-1.0 = -m(3) + c$

$-0.2 = -4m$ $\qquad\qquad$ $-0.5 = -m$

$\Rightarrow 0.05 = m,$ $\qquad\qquad$ $\Rightarrow m = 0.5,$

$0.4 = c$ $\qquad\qquad$ $c = -0.5$

$\Rightarrow y = 0.05x + 0.4$ — ① $\qquad\qquad$ $\Rightarrow y = 0.5x - 0.5$ — ②

$P(0 \leqslant x \leqslant 2.5) = F_x(2.5) - F_x(0)$

$F_x(0) = 0.05(0) + 0.4$ [From ①]

$\Rightarrow 0.4$

$F_x(2.5) = 0.5(2.5) - 0.5$ [From ②]

$\Rightarrow 0.75$

$\Rightarrow P(0 \leqslant x \leqslant 2.5) = 0.75 - 0.4 = \underline{\underline{0.35}}$

8) Find the value of $P(X \geq 1)$.(write your answer correct to two decimal places.)

$P(x \geqslant 1) = 1 - F_x(1)$

$\Rightarrow 1 - (0.05(1) + 0.4)$ [From ①]

$\Rightarrow 1 - 0.45$

$\Rightarrow \underline{\underline{0.55}}$