1) A shopkeeper sells mobile phones. The expected demand for mobile phones is 4 per week. Based on the given information, choose the correct option(s) for the random variable X denoting the number of phones sold in a week.

- [ ] $P(X \geq 10) \geq 0.4$
- [x] $P(X \geq 10) \leq 0.4$
- [ ] $P(X < 10) \geq 0.6$
- [ ] $P(X < 10) \leq 0.6$

Markov's Inequality

$P(X \geqslant a) \leqslant \frac{E[X]}{a}$

$\Rightarrow P(X \geqslant 10) \leqslant \frac{4}{10}$

$\Rightarrow P(X \geqslant 10) \leqslant 0.4$

2) The probability of getting heads while tossing an unfair coin is 0.40. Suppose the coin is flipped 30 times. Find an upper bound using Markov's inequality for the probability that it lands on heads at least 20 times.

$\Rightarrow$ Probability of getting heads $= 0.4$

Coin tossed $= 30$ times

Expectation of heads $= 0.4 \times 30 = 12$

$P(X \geqslant 20) = \frac{E[X]}{20}$

$\Rightarrow \frac{12}{20} = 0.6$

3) Suppose a fair coin is flipped 200 times. Find an upper bound on the probability that the number of times the coin lands on heads is at least 130 or at most 70 using Chebyshev's inequality.

$X \sim \text{Binomial}(200, 1/2)$ [Number of heads]

$E[X] = np = 100$

$Var(X) = np(1-p) = 200(0.5)(1-0.5) = 50 = \sigma^2$

Formula for Chebyshev's inequality

$P(|X-\mu| \geqslant k\sigma) \leqslant \frac{1}{k^2}$

$P(|X-100| \geqslant 30)$

Here, $k\sigma = 30$

$\Rightarrow k\sqrt{50} = 30$

$\Rightarrow k = \frac{30}{\sqrt{50}}$

$\therefore P(|X-100| \geqslant 30) \leqslant \frac{1}{\left(\frac{30}{\sqrt{50}}\right)^2}$

$\Rightarrow \frac{5\cancel{0}}{90\cancel{0}} = \frac{1}{18}$