2) The covariance of two random variables X and Y is 1.5, and variances of X and Y are 1 and 4, respectively. Find $\rho(X, Y)$.

$Cov(X,Y) = 1.5$

$Var(X) = 1 \quad , \quad Var(Y) = 4$

$$\Rightarrow \rho(X,Y) = \frac{Cor(X,Y)}{SD(X) \cdot SD(Y)} = \frac{1.5}{\sqrt{1} \times \sqrt{4}} = \frac{1.5}{2} = 0.75$$

3) Let X and Y be two random variables with joint distribution given in Table 3.6.1.

Y \ X	0	1	2
0	$\frac{1}{12}$	$\frac{3}{12}$	0
1	$\frac{2}{12}$	0	$\frac{1}{12}$
2	$\frac{3}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

Table 3.6.1: Joint distribution of X and Y.

Find $\rho(X, Y)$.

$XY = xy$	0	1	2	4
$P(XY = xy)$	$9/12$	0	$2/12$	$1/12$

$$E[XY] = 0 \times \frac{9}{12} + 1 \times 0 + 2 \times \frac{2}{12} + 4 \times \frac{1}{12} = \frac{8}{12}$$

$$E[X] = 0 \times \frac{6}{12} + 1 \times \frac{4}{12} + 2 \times \frac{2}{12} = \frac{8}{12}$$

$$E[X^2] = 0^2 \times \frac{6}{12} + 1^2 \times \frac{4}{12} + 2^2 \times \frac{2}{12} = \frac{12}{12}$$

$$Var(X) = E[X^2] - E[X]^2 = \frac{12}{12} - \left(\frac{8}{12}\right)^2 = \frac{144-64}{144} = \frac{80}{144}$$

$$E[Y] = 0 \times \frac{4}{12} + 1 \times \frac{3}{12} + 2 \times \frac{5}{12} = \frac{13}{12}$$

$$E[Y^2] = 0^2 \times \frac{4}{12} + 1^2 \times \frac{3}{12} + 2^2 \times \frac{5}{12} = \frac{23}{12}$$

$$Var(Y) = E[Y^2] - E[Y]^2 = \frac{276-169}{144} = \frac{107}{144}$$

$$Cor(X,Y) = \frac{8}{12} - \frac{8}{12} \times \frac{13}{12} \Rightarrow \frac{\cancel{8}^{\cancel{4}^{2}}}{\cancel{12}_{\cancel{6}_{3}}}\left(\frac{-1}{12}\right) = \frac{-1}{18}$$

$$\Rightarrow \rho(X,Y) = \frac{Cor(X,Y)}{\quad} = \frac{-1/18}{\quad}$$

$$\sqrt{Var(X)}\,\sqrt{Var(Y)} \qquad \sqrt{\frac{80}{144}}\;\sqrt{\frac{107}{144}}$$

$$\Rightarrow \frac{-1 \times \overset{\cancel{72}\ 8}{\cancel{144}}}{\underset{\cancel{9}\ 1}{\cancel{18}} \times \sqrt{8560}}$$

$$\Rightarrow \frac{-8}{\sqrt{535 \times 16}} = \frac{-\cancel{8}^{2}}{\cancel{4} \times \sqrt{535}} \Rightarrow \frac{-2}{\sqrt{535}}$$

$$\begin{pmatrix} 2 & 4 & 6 \end{pmatrix} \begin{pmatrix} 1 & -5 & 3 \end{pmatrix} = 2 - 20 + 18$$