2) Let X and Y be two random variables with joint distribution given in Table 3.5.1.

$Y \backslash X$	0	1	2
0	$\frac{1}{12}$	$\frac{3}{12}$	0
1	$\frac{2}{12}$	0	$\frac{1}{12}$
2	$\frac{3}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

Table 3.5.1: Joint distribution of X and Y.

Find $Cov(X, Y)$.

$$cov(x,y) = E[XY] - E[X] . E[Y]$$

$XY = xy$	0	1	2	4
$P(XY = xy)$	$9/12$	0	$2/12$	$1/12$

$$E[XY] = 0 \times \frac{9}{12} + 1 \times 0 + 2 \times \frac{2}{12} + 4 \times \frac{1}{12}$$

$$\Rightarrow 0 + 0 + \frac{4}{12} + \frac{4}{12} = \frac{8}{12}$$

$$E[X] = 0 \times \frac{6}{12} + 1 \times \frac{4}{12} + 2 \times \frac{2}{12}$$

$$\Rightarrow \frac{8}{12}$$

$$E[Y] = 0 \times \frac{4}{12} + 1 \times \frac{3}{12} + 2 \times \frac{5}{12}$$

$$\Rightarrow \frac{13}{12}$$

$$\Rightarrow Cov(XY) = \frac{8}{12} - \frac{8}{12} \times \frac{13}{12}$$

$$\Rightarrow \frac{8}{12}\left(1 - \frac{13}{12}\right) = \left(\frac{\cancel{8}^{\cancel{4}\,2}}{\cancel{12}_{\cancel{6}\,3}}\right)\left(\frac{-1}{12}\right) \Rightarrow \frac{-2}{36} \Rightarrow \frac{-1}{18}$$

3) The probability distribution of a random variable X is given in Table 3.5.2.

$X = x$	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{2}$

Table 3.5.2: PMF of X

Define another random variable $Y = 2X + 3$. Find $Cov(X, Y)$.

X	0	1	2	3
$Y = 2X + 3$	3	5	7	9
$P(Y = 2x + 3)$	$1/8$	$2/8$	$1/8$	$4/8$

$$\Rightarrow E[XY] = 0 \times 3 \times \frac{1}{8} + 1 \times 5 \times \frac{2}{8} + 2 \times 7 \times \frac{1}{8} + 3 \times 9 \times \frac{4}{8}$$

$$\Rightarrow \frac{10}{8} + \frac{14}{8} + \frac{108}{8} = \frac{132}{8}$$

$$E[X] = 0 \times \frac{1}{8} + 1 \times \frac{2}{8} + 2 \times \frac{1}{8} + 3 \times \frac{4}{8}$$

$$\Rightarrow \frac{2}{8} + \frac{2}{8} + \frac{12}{8} = \frac{16}{8}$$

$$E[Y] = 3 \times \frac{1}{8} + 5 \times \frac{2}{8} + 7 \times \frac{1}{8} + 9 \times \frac{4}{8}$$

$$\Rightarrow \frac{3}{8} + \frac{10}{8} + \frac{7}{8} + \frac{36}{8}$$

$$\Rightarrow \frac{56}{8}$$

$$\Rightarrow Cov(X,Y) = \frac{132}{8} - \frac{16}{8} \times \frac{56}{8}$$

$$\Rightarrow \frac{1}{8}\left(132 - \cancel{\frac{16}{8}}^{2} \times 56\right)$$

$$\Rightarrow \frac{20}{8} = \underline{\underline{2.5}}$$

4) The joint distribution of X and Y is given by $f(x,y) = \frac{x+y}{9}$, for $x = 0,1,2; y = 0,1.$

Find $Cov(X,Y)$.

y\x	0	1	2
0	0	1/9	2/9
1	1/9	2/9	3/9

$XY = xy$	0	1	2
$P(XY = xy)$	4/9	2/9	3/9

$$E[XY] = 0 \times \frac{4}{9} + 1 \times \frac{2}{9} + 2 \times \frac{3}{9} = \frac{8}{9}$$

$$E[X] = 0 \times \frac{1}{9} + 1 \times \frac{3}{9} + 2 \times \frac{5}{9} = \frac{13}{9}$$

$$E[Y] = 0 \times \frac{3}{9} + 1 \times \frac{6}{9} = \frac{6}{9}$$

$$\Rightarrow Cov(X,Y) = E[XY] - E[X] \cdot E[Y]$$

$$\Rightarrow \frac{8}{9} - \frac{13}{9} \times \frac{\cancel{6}^{2}}{\cancel{9}_{3}}$$

$$\Rightarrow \quad \frac{8}{9} - \frac{26}{27}$$

$$\Rightarrow \quad \frac{24 - 26}{27} = \frac{-2}{27}$$