

1) If the probability that Nidhi will earn ₹2,000 on any particular day is  $\frac{1}{4}$  and the probability that Nidhi will earn ₹1,500 on the same day is  $\frac{3}{4}$ , what is the variance of the amount that Nidhi will earn on any particular day?

$$E[X] = 2000 \times \frac{1}{4} + 1500 \times \frac{3}{4}$$

$$\Rightarrow 1625$$

$$E[X^2] = (2000)^2 \times \frac{1}{4} + (1500)^2 \times \frac{3}{4}$$

$$\Rightarrow \frac{4 \times 10^6}{4} + \frac{675 \times 10^4}{4}$$

$$\Rightarrow 2687500$$

$$\text{Var}(X) = E[X^2] - E[X]^2$$

$$\Rightarrow 2687500 - 1625^2$$

$$\Rightarrow 46875$$

2) If  $\text{Var}(3X + 4) = 144$ , what is the standard deviation of  $X$ ?

$$\text{Var}(3X + 4) = 144$$

$$\Rightarrow 9 \cdot \text{Var}(X) = 144$$

$$\Rightarrow \text{Var}(X) = 144/9$$

$$\Rightarrow \text{SD}(X) = \sqrt{\text{Var}(X)}$$

$$\Rightarrow \sqrt{\frac{144}{9}}$$

$$\Rightarrow \frac{12}{3} = \underline{\underline{4}}$$

3) If  $X$  and  $Y$  are independent random variables such that  $\text{Var}(X) = \text{Var}(Y) = 9$ , find  $\text{Var}(X - Y)$ .

$$\text{Var}(X - Y) = 1^2 \cdot \text{Var}(X) + (-1)^2 \cdot \text{Var}(Y)$$

$$\Rightarrow \text{Var}(X) + \text{Var}(Y)$$

$$\Rightarrow 9 + 9 = \underline{\underline{18}}$$

4) The annual salary of a certain employee of a company is a random variable with an expected value of ₹600,000 and a standard deviation of ₹40,000. The employee invests 20% of the amount. Find the standard deviation of the amount invested by the employee.

$$20\% \text{ of } 40000 = 8000$$

5) Three fair coins are tossed. Find  $\text{Var}(X)$ , where  $X$  is the number of tails that appear.

$$\text{Sample Space} = \left\{ \begin{array}{l} HHH, \\ HHT, \\ HTH, \\ THH, \\ THT, \\ TTH, \\ HTT, \\ TTT \end{array} \right\}$$

$X = \text{Number of tails}$

$X = n$	0	1	2	3
$P(X=n)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$$E[X] = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$$

$$\Rightarrow \frac{12}{8}$$

$$E[X^2] = 0^2 \times \frac{1}{8} + 1^2 \times \frac{3}{8} + 2^2 \times \frac{3}{8} + 3^2 \times \frac{1}{8}$$

$$\Rightarrow \frac{3 + 12 + 9}{8} = \frac{24}{8}$$

$$\text{Var}(X) = E[X^2] - E[X]^2$$

$$\Rightarrow \frac{24}{8} - \left(\frac{12}{8}\right)^2$$

$$\Rightarrow \frac{24 \times 8}{64} - \frac{144}{64} = \frac{48}{64} = \underline{\underline{0.75}}$$

6) The probability that an event A will occur is 0.60. Define

$$I = \begin{cases} 1 & \text{if event A occurs} \\ 0 & \text{if event A does not occur} \end{cases}$$

Find  $\text{Var}(I)$ .

$$E[X] = (1)(0.6) + (0)(0.4)$$

$$\Rightarrow E[X] = 0.6$$

Similarly,

$$E[X^2] = 0.6$$

$$\Rightarrow \text{Var}(X) = E[X^2] - E[X]^2$$

$$\Rightarrow 0.6 - 0.36$$

$$\Rightarrow \underline{\underline{0.24}}$$