2) Suppose that a random variable X takes values 2, 3, and 4. If $E[X]$ = 3.4 and $P(X = 2) = P(X = 3)$, then find the value of $P(X = 4)$.

$E[X] = 3.4$

Let $P(X=2) = x$

$\Rightarrow P(X=3) = x$

$\Rightarrow P(X=4) = 1-2x$

$\Rightarrow E[X] = 2 \cdot P(X=2) + 3 \cdot P(X=3) + 4 \cdot P(X=4)$

$\Rightarrow 3.4 = 2(x) + 3(x) + 4(1-2x)$

$\Rightarrow 3.4 = 5x + 4 - 8x$

$\Rightarrow \cancel{-}0.6 = \cancel{-}3x$

$\Rightarrow x = 0.2$

$\Rightarrow P(X=4) = 1-2x = 1 - 2(0.2) = \underline{\underline{0.6}}$

Use the below information for Question 3 and 4 Shreya and Ansh works for the same company. Shreya's Diwali bonus is a random variable whose expected value is ₹12,000.

3) If Ansh's bonus is ₹7,000 more than Shreya's, find the expected value of Ansh's bonus.

$E[S] = 12{,}000$ (Shreya's Bonus)

$E[A] = 7000 + E[S]$

$\Rightarrow \underline{\underline{19000}}$

4) If Ansh's bonus is 60% of Shreya's, find the expected value of Ansh's bonus.

$E[A] = \frac{60}{100} E[S]$

$\Rightarrow \frac{3}{5} \times 12{,}000 = \underline{\underline{7200}}$

5) Let X be a discrete random variable with the following probability mass function

$$P(X=k) = \begin{cases} 0.2 & \text{for } k=0 \\ 0.3 & \text{for } k=1 \\ 0.4 & \text{for } k=2 \\ 0.1 & \text{for } k=3 \\ 0 & \text{otherwise.} \end{cases}$$

Define $Y = (X-1)(X-2)$. Find $E[Y]$.

$\Rightarrow Y = (X-1)(X-2)$

$\Rightarrow Y = X^2 - 2X - X + 2$

$\Rightarrow Y = X^2 - 3X + 2$

$\Rightarrow E[Y] = E[X^2] - 3 \cdot E[X] + 2$

$E[X^2] = 0^2 \times 0.2 + 1^2 \times 0.3 + 2^2 \times 0.4 + 3^2 \times 0.1$

$\Rightarrow 0.3 + 1.6 + 0.9$

$\Rightarrow 2.8$

$E[X] = 0 \times 0.2 + 1 \times 0.3 + 2 \times 0.4 + 3 \times 0.1$

$\Rightarrow 0.3 + 0.8 + 0.3$

$\Rightarrow 1.4$

$\Rightarrow E[Y] = 2.8 - 3(1.4) + 2$

$\Rightarrow 2 - 1.4 = \underline{\underline{0.6}}$

The joint PMF of the random variables X and Y is given in Table 3.2.1.

$Y \backslash X$	1	2	3
1	k	k	$2k$
2	$2k$	0	$4k$
3	$3k$	k	$6k$

Table 3.2.1: Joint distribution of X and Y.

6) Find $E[X]$.

$k + k + 2k + 2k + 0 + 4k + 3k + k + 6k = 1$

$\Rightarrow k = \frac{1}{20}$

$E[X] = (1)\left(\frac{6}{20}\right) + (2)\left(\frac{2}{20}\right) + (3)\left(\frac{12}{20}\right)$

$\Rightarrow \frac{6 + 4 + 36}{20} = \frac{\cancel{46}^{23}}{\cancel{20}_{10}} = \underline{\underline{2.3}}$

7) Consider the random variable $Z = X + Y$. Find $E[Z]$.

$E[Z] = E[X] + E[Y]$

$E[Y] = (1)\left(\frac{4}{20}\right) + (2)\left(\frac{6}{20}\right) + (3)\left(\frac{10}{20}\right)$

$\Rightarrow \frac{4 + 12 + 30}{20} = \frac{46}{20} = 2.3$

$\Rightarrow E[Z] = E[X] + E[Y]$

$\Rightarrow 2.3 + 2.3$

$\Rightarrow \underline{\underline{4.6}}$