1) By investing in a particular stock, a person can make a profit of ₹$10,000$ in one year with probability 0.55 or take a loss of ₹$5,000$ with probability 0.45. What is the person's expected gain?

X	$10{,}000$	-5000
$P(X=x)$	0.55	0.45

$$\Rightarrow E[X] = (10{,}000)(0.55) + (-5000)(0.45)$$
$$\Rightarrow 5500 - 2250$$
$$\Rightarrow 3250$$

2) Suppose a random variable X has the following probability distribution:

$X=x$	3	4	5	6	7	8	9
$P(X=x)$	$\frac{1}{12}$	$\frac{1}{24}$	$\frac{1}{6}$	$\frac{1}{24}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{4}$

Table 3.1.1: PMF of X

Find the expected value of X.

$$\Rightarrow E[X] = \frac{1}{12}(3) + \frac{1}{24}(4) + \frac{1}{6}(5) + \frac{1}{24}(6) + \frac{1}{4}(7) + \frac{1}{6}(8) + \frac{1}{4}(9)$$
$$\Rightarrow \frac{3}{12} + \frac{2}{12} + \frac{10}{12} + \frac{3}{12} + \frac{21}{12} + \frac{16}{12} + \frac{27}{12}$$
$$\Rightarrow \frac{82}{12} = \boxed{\frac{41}{6}}$$

3) A coin is biased such that the head is likely to occur three times more than a tail. Find the expected number of heads when the coin is tossed twice independently (Correct upto 1 decimal point).

$X=x$	H	T
$P(X=x)$	$3/4$	$1/4$

Coin is tossed twice independently

$$\Rightarrow E[X] = E[X_1] + E[X_2]$$
$$\Rightarrow 3/4 + 3/4$$
$$\Rightarrow 6/4 = 3/2 = \boxed{1.5}$$

4) The probability distribution of a discrete random variable X is

$$f(x) = {}^4C_x\left(\tfrac{1}{4}\right)^x\left(\tfrac{3}{4}\right)^{4-x}, x = 0, 1, 2, 3, 4.$$

Find the mean of X.

$$E[X] = \underbrace{np}_{\text{Formula for binomial random variable}} = 4 \times \frac{1}{4} = 1$$

5) Suppose that a game is to be played with a single fair die. In the game, a player wins ₹100 if a 3 turns up, ₹150 if a 5 turns up; loses ₹160 if a 4 turns up; while the player neither wins nor loses if any other face turns up. Find the expected money to be won.

$$E[X] = (100).P(X=3) + (150).P(X=5) - (160).P(X=4)$$

$\Rightarrow \quad (100)(1/6) + (150)(1/6) - (160)(1/6)$

$\Rightarrow \quad \frac{1}{6}\left[250 - 160\right]$

$\Rightarrow \quad \frac{\cancel{90}\ 30}{\cancel{6}\ 2} = \boxed{15}$

6) Suppose that a game is to be played with a single biased die. The numbers from two to six are equally likely to land face up, but number one is thrice as likely to land face up as each of the other numbers. In the game, a player wins ₹100 if a 3 turns up, ₹150 if a 5 turns up; loses ₹160 if a 4 turns up; while the player neither wins nor loses if any other face turns up. Find the expected money to be won. (Correct up to 2 decimal points)

$E[X] = (100).P(X=3) + (150).P(X=5) - (160).P(X=4)$

$\Rightarrow \quad \frac{1}{8}\left[250-160\right]$

$\Rightarrow \quad \underline{11.25}$

7) Find the expected value of a discrete random variable X whose PMF is given by

$$f(x) = \frac{1}{16}\,{}^4C_x, x = 0, 1, 2, 3, 4.$$

$E[X] = (0)\left(\frac{1}{16}\cdot{}^4C_0\right) + (1)\left(\frac{1}{16}\cdot{}^4C_1\right) + (2)\left(\frac{1}{16}\cdot{}^4C_2\right) + (3)\left(\frac{1}{16}\cdot{}^4C_3\right) +$

$(4)\left(\frac{1}{16}\cdot{}^4C_4\right)$

$\Rightarrow \quad \frac{1}{16}\left[(1)(4) + (2)(6) + (3)(4) + (4)(1)\right]$

$\Rightarrow \quad \frac{1}{16}\left[4 + 12 + 12 + 4\right]$

$\Rightarrow \quad \frac{32}{16} = \underline{2}$