1) Probability mass function of a random variable is given as:

$$P(X=x)=\begin{cases}0.1 & x=-2\\ 0.3 & x=-1\\ 0.1 & x=0\\ 0.2 & x=1\\ 0.3 & x=2\end{cases}$$

Let $Y=X(X^2-1)(X-2)$

Find the range of Y.

At $x=-2$
$Y = -2((-2)^2-1)(-2-2)$
$Y = -2(3)(-4)$
$Y = 24$

At $x=-1$
$Y = (-1)((-1)^2-1)(-1-2)$
$Y = 0$

Similarly, for all other values of X
$Y = 0$

$\therefore$ Range of $Y = \{0, 24\}$

2) Probability mass function of a random variable is given as:

$$P(X=x)=\begin{cases}0.1 & x=-2\\ 0.3 & x=-1\\ 0.1 & x=0\\ 0.2 & x=1\\ 0.3 & x=2\end{cases}$$

Let $Y=X(X^2-1)(X-2)$.

Find the value of P(Y= 24).

$P(Y=24) = P(X=-2) = 0.1$

3) Let X ~ Geometric(p) and

$$f(x)=\begin{cases}\frac{x}{2} & \text{if } x \text{ is an even number}\\ \frac{x+1}{2} & \text{if } x \text{ is an odd number}\end{cases}$$

Find the range of $f(X)$.

Note: Geometric variables dont take values from 0 as you need atleast 1 trial to get a success.

for $X=1$
$f(x) = \frac{1+1}{2} = 1$

for $X=2$
$f(x) = \frac{2}{2} = 1$

for $X=3$
$f(x) = \frac{3+1}{2} = 2$

$\Rightarrow$ Range of $X = \{1, 2, 3 \ldots\}$

4) Let $X \sim$ Geometric(p) and

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } x \text{ is an even number} \\ \frac{x+1}{2} & \text{if } x \text{ is an odd number} \end{cases}$$

Find the probability that $f(X) = k$
where k is in the range of $f(X)$.

when x is an even number,

$f(x) = \frac{x}{2}$

$\Rightarrow 2 \cdot f(x) = x$ $\quad [f(x) = k]$

$\Rightarrow x = 2k$

$\Rightarrow P(X = 2k) = (1-p)^{x-1} \cdot p$

$\Rightarrow (1-p)^{2k-1} \cdot p$

when x is an odd number,

$f(x) = \frac{x+1}{2}$

$\Rightarrow 2 \cdot f(x) - 1 = x$ $\quad [f(x) = k]$

$\Rightarrow x = 2k-1$

$\Rightarrow P(X = 2k-1) = (1-p)^{x-1} \cdot p$

$\Rightarrow (1-p)^{2k-1-1} \cdot p$

$\Rightarrow (1-p)^{2k-2} \cdot p$

$\Rightarrow$ X can be even or X can be odd

$\Rightarrow P(X = 2k) + P(X = 2k-1)$

$\Rightarrow (1-p)^{2k-1} \cdot p + (1-p)^{2k-2} \cdot p$

$\Rightarrow p\left[(1-p)^{2k-1} + (1-p)^{2k-2}\right]$

$\Rightarrow p\left[(1-p)^{2k-2} \cdot (1-p) + (1-p)^{2k-2}\right]$

$\Rightarrow (1-p)^{2k-2} \cdot p\left[1-p+1\right]$

$\Rightarrow (1-p)^{2(k-1)} \cdot p\left[2-p\right]$