1) Choose the correct statements from the following:

- [x] If X_1, X_2, and X_3 are independent random variables, $f_{X_1 X_2 | X_3=(t_3)}(t_1, t_2) = f_{X_1}(t_1) f_{X_2}(t_2)$. (Check Formula)
- [ ] If $X_, X_2$, andX_3 are independent random variables, $f_{X_1 | X_3=t_3}(t_1) = f_{X_1}(t_1) f_{X_2}(t_2)$.
- [ ] If $X_1, X_2, \cdots, X_n$ are independent random variables, X_1, X_2, and X_3 need not be independent.
- [x] If $X_1, X_2, \cdots, X_n$ are independent, any subset of X_i will also be independent. (Fact)

2) Let $X_1, X_2, \cdots, X_{10} \sim i.i.d.$ Geometric($\frac{1}{5}$). Find the probability that $(X_1 > 10, X_2 > 10, \cdots, X_{10} > 10)$.

$X \sim \text{Geometric}\left(\frac{1}{5}\right)$

$P(X > x) = 1 - P(X \leq x)$

$\Rightarrow 1 - \underbrace{\left[1 - (1-p)^x\right]}$

This is a formula for geometric random variables.

$\Rightarrow (1-p)^x$

$\Rightarrow \left(1 - \frac{1}{5}\right)^{10}$ $\quad (p = 1/5 \,,\; x = 10)$

$\Rightarrow (0.8)^{10}$

$\Rightarrow P(X_1 > 10,\ X_2 > 10,\ X_3 > 10,\ \ldots\ X_{10} > 10)$

$\Rightarrow$ As $X_1, X_2 \ldots X_{10} \sim$ i.i.d (independent variables)

$\Rightarrow P(X_1 > 10) \times P(X_2 > 10) \times P(X_3 > 10) \ldots P(X_{10} > 10)$

$\Rightarrow (0.8)^{10} \times (0.8)^{10} \times (0.8)^{10} \ldots (0.8)^{10}$

$\Rightarrow (0.8)^{100}$

3) A random experiment consists of rolling a fair die until six appears. Let X denote the number of times the die is rolled. Suppose six does not appear until the sixth throw, find the probability that six will appear after the eighth throw of die. **1 point**

- $(\frac{1}{6})^2$
- $(\frac{5}{6})^2$
- $(\frac{1}{6})^8$
- $(\frac{5}{6})^8$

$X \sim \text{Geometric}\left(\frac{1}{6}\right)$

Find: $P(X > 8 \mid X > 6)$

$\Rightarrow P(X > 6+2 \mid X > 6)$

$\Rightarrow$ Using memoryless property

$\Rightarrow P(X > 6 + 2 \mid X > 6) = P(X > 2)$

$\Rightarrow P(X > 2) = 1 - P(X \leq 2)$

$\Rightarrow 1 - \left[1 - (1 - 1/6)^2\right]$

$\Rightarrow \left(\frac{5}{6}\right)^2$