1. Let $X_1, X_2, X_3, X_4 \sim f_{X_1X_2X_3X_4}(t_1, t_2, t_3, t_4)$. Their joint PMF is given in Table 2.10.1.

t_1	t_2	t_3	t_4	$f_{X_1X_2X_3X_4}(t_1,t_2,t_3,t_4)$
0	1	0	0	1/5
0	0	1	0	1/5
0	0	0	1	1/5
0	0	1	1	1/5
0	1	1	1	1/5

Table 2.10.1: Joint PMF of X_1, X_2, X_3 and X_4.

Define $g(X_1, X_2, X_3, X_4) = X_1 + X_2 + X_3X_4$ and $h(X_1, X_2, X_3, X_4) = X_2X_3X_4$.

1) Determine the range of g and h.

Check for all the combinations row-wise.

Range of $g = \{0, 1, 2\}$

Range of $h = \{0, 1\}$

3) A random experiment consists of rolling an unbiased die two times. Let X denote the number obtained on the first die and Y denote the number obtained on the second die. The joint PMF of X and Y is **1 point** denoted by $f_{XY}(t_1, t_2)$. Let $Z = X + Y$. Choose the correct statements from the following:

- [x] $P(Z=3) = \sum_{x=1}^{2} f_{XY}(x, 3-x)$ → $\boxed{Z = X + Y \quad 3 = X + Y \quad Y = 3 - X}$ x goes from 1 to 2 only because Z is given as 3.
- [ ] $P(Z=3) = \sum_{x \in T_X, y \in T_Y} f_{XY}(3-x, 3-y)$
- [ ] Z is uniformly distributed over its range.
- [x] $f_Z(z) = \sum_{x=1}^{6} f_X(x) f_Y(z-x)$ → $\boxed{Z = X + Y \quad Y = Z - X}$

Let the random variables X and Y, which represent the number of people visiting shopping malls in city 1 and city 2 in an one hour interval, respectively, follow the Poisson distribution. The average number of people visiting the shopping malls in city 1 and city 2 is 10 per hour and 20 per hour, respectively. Assume that X and Y are independent.

4) Let Z denote the total number of people visiting shopping malls in city 1 and city 2. Find the pmf of Z, $f_Z(z)$. **1 point**

$Z = X + Y$

$$\Rightarrow f_Z(z) = \frac{e^{-\lambda} \cdot \lambda^z}{z!}$$

$$= \frac{e^{-30} \cdot 30^z}{z!} \qquad \left(\begin{array}{l}\lambda = \lambda_x + \lambda_y \\ \lambda = 10 + 20\end{array}\right)$$

5) Find the conditional distribution of Y given that the total number of people visiting shopping malls in city 1 and city 2 is 30.

$$P(Y \mid Z = 30) \sim \text{Binomial}\left(30, \frac{\lambda_2}{\lambda_1 + \lambda_2}\right)$$

$$\Rightarrow \text{Binomial}\left(30, \frac{20}{10+20}\right)$$

$$\Rightarrow \text{Binomial}\left(30, \frac{2}{3}\right)$$