1) Let $X, Y \sim f_{XY}(t_1, t_2)$. Choose the correct statements from the following:

- [x] If $f_{XY}(t_1,t_2) \neq f_X(t_1)f_Y(t_2)$, for some $t_1 \in T_X, t_2 \in T_Y$, then X and Y are dependent.
- [ ] $f_{XY}(t_1,t_2) = f_X(t_1)f_Y(t_2)$, for some $t_1 \in T_X, t_2 \in T_Y$ implies X and Y are independent.
- [x] $f_{XY}(t_1,t_2) = f_X(t_1)f_Y(t_2)$, for all $t_1 \in T_X, t_2 \in T_Y$ implies X and Y are independent.
- [x] If X and Y are independent, $f_{Y|X=t_1}(t_2) = f_Y(t_2)$.

$$f_{XY}(t_1, t_2) = f_X(t_1) \cdot f_Y(t_2)$$

if the condition isn't satisfied for any of the X and Y, then X and Y are dependent

2) Let X_1 and X_2 be two discrete random variables with joint PMF $f_{X_1X_2}(t_1,t_2)$ given in Table 2.1.1. Identify the correct statements from the following:

t_2 \ t_1	1	2	3
1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{16}$
2	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$
3	k	$\frac{1}{16}$	$\frac{1}{16}$

Table 2.1.1: Joint PMF of X_1 and X_2.

- [x] $f_{X_1X_2}(1,2) = f_{X_1}(1)f_{X_2}(2)$. a)
- [ ] $f_{X_1X_2}(1,3) = f_{X_1}(1)f_{X_2}(3)$. b)
- [ ] X_1 and X_2 are independent.
- [x] X_1 and X_2 are not independent.

$$k = 1 - \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{16} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} + \frac{1}{16}\right)$$

$$\Rightarrow k = 1 - \left(\frac{2}{4} + \frac{\cancel{4}^{1}}{\cancel{16}_{4}} + \frac{\cancel{2}^{1}}{\cancel{8}_{4}}\right)$$

$$\Rightarrow k = 1 - 1 = 0$$

if $k = 0$,

$\Rightarrow$ b) is obviously wrong.

$$f_{X_1}(1) = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} \quad \Big| \quad f_{X_2}(2) = \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{2}{4}$$

$$f_{X_1X_2}(1,2) = \boxed{\frac{1}{4}} \quad \Big| \quad f_{X_1}(1) \cdot f_{X_2}(2) = \frac{2}{4} \times \frac{2}{4} = \boxed{\frac{1}{4}}$$

3) Let X and Y be two independent random variables such that $f_{X|Y=t_2}(t_1) = 0.02$ and the marginal PMF $f_Y(t_2) = 0.6$, for $t_2 \in T_Y$. Calculate $f_X(t_1)$.

$f_X(t_1)$ will be same as $f_{X|Y=t_2}(t_1)$ because X and Y are independent.

$$\Rightarrow f_X(t_1) = f_{X|Y=t_2}(t_1) = 0.02$$

4) Let the random variables X and Y be independent and let they take values in $\{1,2,3\}$ and $\{1,2\}$, respectively. If $f_X(1) = f_X(3) = \frac{1}{4}$ and $f_Y(2) = \frac{2}{5}$, which of the following can be the joint pmf of X and Y ? ***1 point***

Given,

$f_X(1) = 1/4$
$f_X(3) = 1/4$
| $f_Y(2) = 2/5$

$$\Rightarrow f_{XY}(1,2) = f_X(1) \cdot f_Y(2) = 2/20 = 1/10$$

$$\Rightarrow f_{XY}(3,2) = f_X(3) \cdot f_Y(2) = 2/20 = 1/10$$

Sum of the first and third columns should be = 1/4
Sum of the second row should be = 2/5

Use these checks to identify the correct answer.