

1. Let $X_1, X_2, X_3, X_4 \sim f_{X_1, X_2, X_3, X_4}(t_1, t_2, t_3, t_4)$. Their joint PMF is given in Table 1.7.1.

t_1	t_2	t_3	t_4	$f_{X_1, X_2, X_3, X_4}(t_1, t_2, t_3, t_4)$
0	0	0	0	1/8
0	1	0	0	1/8
0	1	1	0	1/8
0	0	0	3	1/8
1	0	1	2	1/8
1	1	1	1	1/8
1	1	1	0	1/8
1	1	1	1	1/8

Table 1.7.1: Joint PMF of X_1, X_2, X_3 and X_4 .

1) Choose the correct options from the following:

- $T_{X_1} = T_{X_2} = T_{X_3} = \{0, 1\}$ and $T_{X_4} = \{0, 1, 2, 3\}$.
- $T_{X_1} = T_{X_2} = \{0, 1\}, T_{X_3} = \{0\}$ and $T_{X_4} = \{0, 1, 2, 3\}$.
- Range of $(X_1 | X_2 = 0)$ is $\{0\}$.
- Range of $(X_2 | X_1 = 0, X_3 = 1)$ is $\{1\}$.

Q) Calculate $f_{X_1 | X_2=0, X_4=0}(1)$

$$\Rightarrow \frac{P(X_1=1, X_2=0, X_4=0)}{P(X_2=0, X_4=0)}$$

$$\Rightarrow \frac{0}{P(X_2=0, X_4=0)} = 0$$

Q) Calculate $P(X_3=1, X_4=1 | X_1=1, X_2=1)$

$$\Rightarrow \frac{P(X_1=1, X_2=1, X_3=1, X_4=1)}{P(X_1=1, X_2=1)}$$

$$\Rightarrow \frac{2/8}{3/8} = \frac{2}{3}$$