The number of customers arriving at a shopping centre in an one hour interval is $N \sim$ Poisson(20). Assume that the probability of customer making a purchase is 0.4, and that choice of purchase of customers is independent. Let X be the number of customers who are making a purchase.

1) Identify Range of $(X|N=n)$

- ○ $T_{(X|N=n)} = \{1, 2, \cdots, n\}$
- ○ $T_{(X|N=n)} = \{1, 2, \cdots \infty\}$
- ✓ $T_{(X|N=n)} = \{0, 1, 2, \cdots, n\}$
- ○ $T_{(X|N=n)} = \{0, 1, 2, \cdots, \infty\}$

min no. of customers = 0
max no. of customers = n

2) Identify distribution of $(X|N=n)$

$\Rightarrow P(X|N=n) = \text{Binomial}(n, 0.4)$

3) Choose correct options for values taken by the joint distribution of X and N.

- ☐ $f_{NX}(10, 2) = \dfrac{e^{-20}20^{10}(0.4)^2(0.6)^8}{2!8!}$
- ☐ $f_{NX}(10, 2) = \dfrac{e^{-20}20^{10}(0.6)^2(0.4)^8}{2!8!}$
- ☐ $f_{NX}(10, 12) = 0$
- ☐ $f_{NX}(10, 12) = \dfrac{e^{-20}8^{20}}{2!8!}$

$$f_{X/N=n}(x) = \frac{f_{NX}(N=n, X=x)}{f_N(n)}$$

$$\Rightarrow f_{NX}(n, x) = f_N(n) \cdot f_{X|N=n}(x) = \left(\frac{e^{-\lambda} \cdot \lambda^n}{n!}\right)\left({}^nC_x (0.4)^x (0.6)^{n-x}\right)$$

$$\Rightarrow f_{NX}(10, 2) = \left(\frac{e^{-20} \cdot 20^{10}}{\cancel{10!}}\right)\left(\frac{\cancel{10!}}{2!\,8!} (0.4)^2 (0.6)^8\right)$$

$$\Rightarrow \frac{\left(e^{-20} \cdot 20^{10}\right)\left((0.4)^2 (0.6)^8\right)}{2!\,8!}$$

$f_{NX}(10, 12) = 0$ (its impossible to choose 12 customers from 10 customers)

4) Find the distribution of X.

$\Rightarrow X \sim \text{Poisson}(p\lambda)$

$\Rightarrow X \sim \text{Poisson}(0.4 \times 20)$

$\Rightarrow X \sim \text{Poisson}(8)$

Let $X \sim$ Uniform($\{1, 2\}$) and let Y be the number of aces obtained from a deck of well shuffled 52 cards in X draws (with replacement)

5) Choose the correct statements from the following: ***1 point***

- [x] Range of $(Y|X = 2) = 0, 1, 2.$ Out of 2 cards, possible number of Aces $(0, 1, 2)$
- [ ] Range of $(Y|X = 1) = 0, 1, 2.$
- [x] $(Y|X = 1) \sim$ Binomial $(1, \frac{1}{13})$ Probability of Ace $= \frac{4}{52} = \frac{1}{13}$, no. of cards $= n = 1$
- [ ] $(Y|X = 2) \sim$ Binomial $(1, \frac{1}{13})$

6) Identify the joint PMF of X and Y from the following:

$Y \backslash X$	1	2
0	$\frac{6}{13}$	$\frac{72}{13^2}$
1	$\frac{1}{26}$	$\frac{12}{13^2}$
2	0	$\frac{1}{338}$