1) From a well shuffled deck of 52 cards, four cards are selected at random. Let the random variable X denote the number **1 point** of queens drawn, and let the random variable Y denote the number of kings drawn. Find $f_{X|Y=1}(2)$.

$$f_{X|Y=1}(2) = \frac{P(X=2, Y=1)}{P(Y=1)}$$

$$\Rightarrow P(X=2, Y=1) = \frac{4}{52} \times \frac{3}{51} \times \frac{4}{50} \times \frac{44}{49} \times \frac{4!}{2!}$$

$$P(Y=1) = \frac{4}{52} \times \frac{47}{51} \times \frac{46}{50} \times \frac{45}{49} \times \boxed{\frac{4!}{3!}}$$

$4! = $ Number of ways 4 items can be arranged

$3! = $ Number of common cards

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2) $X, Y \sim f_{XY}(t_1, t_2)$. Identify the correct statements from the following:

○ Range of $(Y|X = t_1)$ will always be same as range of Y.

○ Range of $(X|Y = t_2)$ will always be same as range of Y.

☑ Range of $(Y|X = t_1)$ can be different from range of Y.

○ Range of $(X|Y = t_2)$ will be same as range of X.

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3) $X, Y \sim f_{XY}(t_1, t_2)$. Choose the correct options from the following:

☑ $f_{XY}(t_1, t_2) = f_{X|Y=(t_2)}(t_1) f_Y(t_2)$

☑ $f_{XY}(t_1, t_2) = f_{Y|X=(t_1)}(t_2) f_X(t_1)$

☐ $f_{XY}(t_1, t_2) = f_{Y|X=(t_1)}(t_2) f_Y(t_2)$

☐ $\sum_{t_2 \in T_y} f_{Y|X=(t_1)}(t2) = 1$

Let X and Y be two random variables with joint PMF $f_{XY}(t_1, t_2)$ given in Table 1.3.1.

t_2 \ t_1	1	2	3
1	0	0.10	0.08
2	0.20	0.10	0
3	0.02	0.30	0.20

4) Find the range of $(Y|X = 1)$.

5) Calculate $f_{X|Y=2}(1)$.

$$f_{X|Y=2}(1) = \frac{f_{XY}(1,2)}{f_Y(2)} = \frac{0.20}{0.20+0.10} = \frac{2}{3}$$