1) An experiment consists of rolling an unbiased die two times. The random variables $X_i \sim$ Uniform$\{1,2,3,4,5,6\}$, represent the number on the $i^{th}roll, i = 1, 2$. Calculate $f_{X_1X_2}(3,2)$.

- ○ $\frac{1}{6}$
- ○ $\frac{1}{12}$
- ◉ $\frac{1}{36}$
- ○ $\frac{1}{18}$

$f_{X_1X_2}(2,3) = f_{X_1}(2) \cdot f_{X_2}(3)$

$\Rightarrow \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

2) The joint probability mass function of two discrete random variables X and Y is given in Table 1.1.1. Which of the following are possiable values of a and b? **1 point**

Y \ X	0	1	2	3
1	$\frac{1}{24}$	$\frac{2}{24}$	$\frac{3}{24}$	$\frac{4}{24}$
2	0	$\frac{1}{24}$	a	b
3	0	0	$\frac{1}{24}$	0

Table 1.1.1: Joint PMF of X and Y.

If PMF

$\Rightarrow$ Sum of probabilities $= 1$

$\Rightarrow \frac{1}{24} + \frac{2}{24} + \frac{3}{24} + \frac{4}{24} + \frac{1}{24} + a + b + \frac{1}{24} = 1$

$\Rightarrow a + b = 1 - \frac{12}{24}$

$\Rightarrow \boxed{a + b = \frac{1}{2}}$

3) From a well shuffled deck of 52 cards, four cards are selected at random. Let the random variable X denote the number of queens drawn, and let the random variable Y denote the number of kings drawn. Find $f_{XY}(2,1)$. **1 point**

Total no. of kings $= 1 \times 4 = 4$

" " " queens $= 1 \times 4 = 4$

$\Rightarrow f_{XY}(2,1) = \frac{4}{52} \times \frac{3}{51} \times \frac{4}{50} \times \frac{44}{49} \times \frac{4!}{2!}$

$4! =$ number of ways 4 cards can be arranged

$2! =$ number of common cards (2 Queens)

The joint probability mass function of two discrete random variables X and Y is given by

$g_{XY}(x,y) = \frac{xy}{9}, \quad x, y \in \{1,2\}$.

4) Find $P(X + Y = 1)$.

$P(X + Y = 1) = 0$

Min values x and y can take $= 1, 1$

$\Rightarrow$ $X + Y$ will always be greater than or equal to 2.

5) Find $P(X - Y = 1)$.

$$P(X - Y = 1) = g_{XY}(2,1)$$

$$\Rightarrow \frac{2}{9}$$